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Understanding the masses of the $c\bar{s}$ states in Regge phenomenology

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Abstract. In the framework of Regge phenomenology, the masses of the charmed states $c\bar{q}$ ($q = u, d, s$) lying on the $1\,^3S_1$-like trajectories are estimated. The overall agreement between our estimated masses and the recent predictions given using modified quark models by Matsuki et al., Lakhina et al. and Close et al. is good. The masses of the observed charmed states $D_{s0}(2317)$, $D_{sJ}(2860)$ and $D_{sJ}(2690)/D_{sJ}(2700)$ can reasonably be reproduced in the picture of these charmed states as simple quark–antiquark configurations. We therefore suggest that $D_{s0}(2317)$ can be identified with the $c\bar{s}(1\,^3P_0)$ states. The possible assignments of the $D_{sJ}(2860)$ and $D_{sJ}(2690)/D_{sJ}(2700)$ are discussed.

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1 Introduction

Recently, the discovery of the new charm–strange state $D_{s0}(2317)$ [1–4] generated strong interest in charmed meson spectroscopy. $D_{s0}(2317)$ seems to be an obvious candidate for the $1\,^3P_0$ $c\bar{s}$ state, since it is the first observed charm–strange 0^+ resonance. However, the observed mass of the $D_{s0}(2317)$ is more than one hundred MeV higher than the constituent quark model predictions for $1\,^3P_0$ $c\bar{s}$. For example, the measured mass of $D_{s0}(2317)$ is 2317.3 ± 0.6 MeV [5], while the prediction for the $1\,^3P_0$ $c\bar{s}$ state by Isgur and Godrey is 2.48 GeV [6], and that by Di Pierro and Eichten is 2.487 GeV [7]. It is widely accepted that the constituent quark model offers the most complete description of the hadron properties and is probably the most successful phenomenological model for the structure of the hadron [8]. Therefore, the substantially small observed mass of $D_{s0}(2317)$ led to many exotic interpretations of the underlying structure of $D_{s0}(2317)$: the models of a (DK) molecule, a four-quark state, a $D\pi$ atom or baryonium have been proposed in the literature. (For detailed reviews, see e.g. [9–12].)

It should be noted that it is very important to exhaust the possible conventional $c\bar{s}$ descriptions of $D_{s0}(2317)$ before resorting to more exotic models, and the discrepancy between the measured mass of $D_{s0}(2317)$ and the quark model predictions for the $1\,^3P_0$ $c\bar{s}$ mass maybe imply that the approximations to the constituent quark model are not appropriate. In fact, it has been pointed out by Matsuki et al. [13] that conventional quark models do not completely and consistently respect the heavy quark symmetry and the $D_{s0}(2317)$ mass can be reproduced if one treats a bound state equation appropriately. Also, it is found by Lee et al. [14] that with one loop chiral corrections, the quark model for the $c\bar{s}$ mesons can naturally account for the unusual mass of $D_{s0}(2317)$. More recently, it has been shown that a simple modification to the standard vector Coulomb plus scalar linear quark potential model maintains good agreement with the charmonium spectrum and agrees remarkably well with the D and D_s spectra [15, 16].

In the present work, we shall show in Regge phenomenology that the masses of the recently observed charmed states $D_0(2290)$, $D_{s0}(2317)$, $D_{sJ}(2860)$ [17] and $D_{sJ}(2690)/D_{sJ}(2700)$ [17, 18] can be reproduced within a simple $c\bar{q}$ picture, and our estimated masses are in good agreement with those predicted by [13, 15, 16]. Therefore, our analysis supports the conclusion that it is not necessary to introduce more exotic models for understanding the masses of these reported charmed states.

2 Regge phenomenology

Regge theory is concerned with the particle spectrum, the forces between particles, and the high energy behavior of the scattering amplitudes [19]. One of the most distinctive features of Regge theory is the Regge trajectory by which the mass and the spin of a hadron are related. Knowledge of the Regge trajectories is useful not only for the spectral purposes, but also for many non-spectral purposes. The intercepts and slopes of the Regge trajectories are of fundamental importance in hadron physics [20].

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A series of recent papers by Anisovich et al. [21–24] show that the meson states fit to the quasi-linear Regge trajectories with sufficiently good accuracy, although some suggestions exist that realistic Regge trajectories could be nonlinear [25–31].

The quasi-linear Regge trajectories for a meson multiplet can be parameterized as follows:

$$J = \alpha_N(t) = \alpha_{i\bar{i}'(N)}(0) + \alpha'_{i\bar{i}'(N)} t \,, \qquad (1)$$

where i ($\bar{i}'$) refers to the quark (antiquark) flavor, $i\bar{i}'(N)$ denotes the meson $i\bar{i}'$ with radial quantum number N ($N = 1, 2, 3, \dots$), $t = M^2_{i\bar{i}'(N)}$, and J and $M_{i\bar{i}'(N)}$ are respectively the spin and mass of the $i\bar{i}'(N)$ meson; $\alpha_{i\bar{i}'(N)}(0)$ and $\alpha'_{i\bar{i}'(N)}$ are respectively the intercept and slope of the trajectory on which the $i\bar{i}'(N)$ meson lies. For a meson multiplet, the parameters for different flavors can be related by the following relations (see [32] and references therein):

(i) additivity of intercepts,

$$\alpha_{i\bar{i}(N)}(0) + \alpha_{j\bar{j}(N)}(0) = 2\alpha_{i\bar{j}(N)}(0) \,, \qquad (2)$$

(ii) additivity of inverse slopes,

$$\frac{1}{\alpha'_{i\bar{i}(N)}} + \frac{1}{\alpha'_{j\bar{j}(N)}} = \frac{2}{\alpha'_{i\bar{j}(N)}} \,. \qquad (3)$$

From (1) and (2), we obtain

$$M^2_{i\bar{i}(N)}\alpha'_{i\bar{i}(N)} + M^2_{j\bar{j}(N)}\alpha'_{j\bar{j}(N)} = 2M^2_{i\bar{j}(N)}\alpha'_{i\bar{j}(N)} \,. \qquad (4)$$

The main purpose of this work is to discuss whether the masses of the recent observed charmed states such as $D_0(2290)$, $D_{s0}(2317)$, $D_{sJ}(2860)$ and $D_{sJ}(2690)$/$D_{sJ}(2700)$ can be reproduced correctly in the conventional quark–antiquark picture based on Regge phenomenology. Because the possible quantum numbers of $D_{sJ}(2860)$ include 0^+, 1^-, 2^+ and 3^- [17], J^P of $D_{sJ}(2700)$ ($D_{sJ}(2690)$) is 1^- [18][1], and the 0^+, 1^-, 2^+ and 3^- trajectories are the parity partners [31]; in the following, we shall adopt two assumptions: (a) the slopes of the parity partners' trajectories coincide, as proposed by [31], and (b) $\alpha'_{i\bar{j}(N)} = \alpha'_{i\bar{j}(1)}$, a value adopted by [21–24]. Under these two assumptions, 0^+, 1^-, 2^+ and 3^- are the $1\,^3S_1$-like trajectories. In this work, the slopes of the $1\,^3S_1$-like trajectories used as input are taken from our previous work [32], and these slopes are shown inTable 1. In the following, n denotes a u or d quark.

With the help of $\alpha'_{i\bar{j}(N)} = \alpha'_{i\bar{j}(1)}$, from (1) one has

$$M^2_{i\bar{j}(N)} - M^2_{i\bar{j}(1)} = \frac{\alpha_{i\bar{j}(1)}(0) - \alpha_{i\bar{j}(N)}(0)}{\alpha'_{i\bar{j}(1)}} \,. \qquad (5)$$

[1] In the D^0K^+ system, BaBar observed a resonance (D_{sJ} (2690)) with a mass $2688 \pm 4 \pm 2$ MeV and width $112 \pm 7 \pm 36$ MeV [17]; Belle subsequently observed a resonance (D_{sJ} (2670)) with a mass $2715 \pm 11^{+11}_{-14}$ MeV, width $115 \pm 20^{+36}_{-32}$ MeV and $J^P = 1^-$ [18]. We regard these compatible, and $D_{sJ}(2690)$ and $D_{sJ}(2700)$ should be the same resonance with $J^P = 1^-$.

Table 1. Slopes of the $1\,^3S_1$-like trajectories taken from [32], all in GeV^{-2}

$\alpha'_{n\bar{n}(1)}$	$\alpha'_{n\bar{s}(1)}$	$\alpha'_{c\bar{c}(1)}$	$\alpha'_{c\bar{n}(1)}$	$\alpha'_{c\bar{s}(1)}$
0.8830	0.8493	0.4364	0.5841	0.5692

If $\alpha_{i\bar{j}(1)}(0) - \alpha_{i\bar{j}(N)}(0)$ is simplified to be flavor-independent, (5) can be reduced to the mass formula presented by Anisovich [21–24][2]:

$$M^2_{i\bar{j}(N)} - M^2_{i\bar{j}(1)} = \frac{N-1}{\alpha'_{i\bar{j}(1)}} \,. \qquad (6)$$

Setting $\alpha'_{c\bar{c}(1)} = 0.4364\,\mathrm{GeV}^{-2}$ and $M_{J/\psi} = 3096.916$ MeV, the masses of the $2\,^3S_1$, $3\,^3S_1$ and $4\,^3S_1$ $c\bar{c}$ states predicted by relation (6) are respectively (in MeV) 3447, 3764 and 4058, which are several hundreds MeV lower than the corresponding measured masses [5] (in MeV) 3686.093 ± 0.034, 4039 ± 1 and 4421 ± 4. This implies that the simplification that $\alpha_{i\bar{j}(1)}(0) - \alpha_{i\bar{j}(N)}(0)$ is flavor-independent may be too rough for the heavy mesons. The phenomenological analysis indicates [33, 34] that $\alpha_{i\bar{j}(1)}(0) - \alpha_{i\bar{j}(N)}(0)$ depends on the masses of the constituent quarks, m_i and m_j, and the functional dependence of $\alpha_{i\bar{j}(1)}(0) - \alpha_{i\bar{j}(N)}(0)$ on the quark masses is through the combination $m_i + m_j$. Furthermore, the quantitative results of [33, 34] show $\alpha_{i\bar{j}(1)}(0) - \alpha_{i\bar{j}(2)}(0) \approx 1.3$–$1.6$. This idea motivates us to introduce a factor $(1 + f_{ij}(m_i + m_j))$ into relation (6) for incorporating the corrections due to the flavor-dependent spacing between $\alpha_{i\bar{j}(1)}(0)$ and $\alpha_{i\bar{j}(N)}(0)$:

$$M^2_{i\bar{j}(N)} - M^2_{i\bar{j}(1)} = \frac{(N-1)}{\alpha'_{i\bar{j}(1)}}(1 + f_{ij}(m_i + m_j)) \,, \qquad (7)$$

where the parameter f_{ij} depends on the flavors i and j, and it can be obtained by fitting to the data; the masses of the constituent quarks are taken to have the following values (in GeV):

$$m_u = m_d = 0.29 \,, \quad m_s = 0.46 \,, \quad m_c = 1.65 \,, \qquad (8)$$

which are typical values used in phenomenological quark models [35, 36].

Inserting the values of the J/ψ ($1\,^3S_1$ $c\bar{c}$, mass, 3096.916 MeV), $\psi(4040)$ ($3\,^3S_1$ $c\bar{c}$, 4039 MeV), ρ ($1\,^3S_1$ $n\bar{n}$, 775.5 MeV), $\rho(1450)$ ($2\,^3S_1$ $n\bar{n}$, 1459 MeV), $K^*(892)$ ($1\,^3S_1$ $n\bar{s}$, 896 MeV) and finally $K^*(1580)$ ($2\,^3S_1$ $n\bar{s}$, 1580 MeV)[3]

[2] It is expected that trajectories may occur in integrally spaced sequences, with a 'parent' trajectory $\alpha_1(t)$, and an infinite sequence of 'daughters' $\alpha_N(t) = \alpha_1(t) - n_r$, $n_r = 0, 1, 2, 3, \ldots$, $N = n_r + 1$ [19]. In [21–24], the mass relation $M^2_N = M^2_0 + (N-1)\mu^2$, $\mu^2 = 1/\alpha'$ is presented.

[3] All the masses used as input are taken from PDG [5] except for the $2\,^3S_1$ kaon mass. The assignment of $K^*(1410)$ to the $2\,^3S_1$ kaon is problematic [37, 38]. The quark model and other phenomenological approaches consistently suggest that the $2\,^3S_1$ kaon has a mass of about 1580 MeV [39]; here we take 1580 MeV as the mass of $2\,^3S_1$ $n\bar{s}$.

Table 2. Masses of $c\bar{c}$ states lying on the $1\,^3S_1$-like trajectories. Boldface values stand for masses used as input. All in GeV

$c\bar{c}$	Expt. [5]	This work	[6]	[40]	[41]	[42]	[43]
$1\,^3S_1$	3.097	**3.097**	3.10	3.10	3.096	3.100	3.15
$2\,^3S_1$	3.686	3.600	3.68	3.73	3.686	3.676	3.63
$3\,^3S_1$	4.039	**4.039**	4.10	4.18	4.088	4.079	4.04
$4\,^3S_1$	4.421	4.434	4.45	4.56		4.434	4.42
$1\,^3D_1$	3.771	**3.771**	3.82	3.80	3.798	3.794	3.76
$2\,^3D_1$	4.153	4.192	4.19	4.22		4.156	4.17
$3\,^3D_1$		4.576	4.52	4.59		4.482	4.53
$4\,^3D_1$		4.929				4.889	4.87
$1\,^3P_0$	3.415	**3.415**	3.42	3.44	3.434	3.412	3.42
$2\,^3P_0$		3.875	3.92	3.94	3.854	3.867	3.86
$3\,^3P_0$		4.287				4.228	4.25
$4\,^3P_0$		4.662				4.538	4.61
$1\,^3D_3$		3.765	3.85	3.83	3.815		
$2\,^3D_3$		4.187	4.22	4.24			
$3\,^3D_3$		4.571					
$4\,^3D_3$		4.924					
$1\,^3P_2$	3.556	**3.556**	3.55	3.54	3.556	3.552	3.56
$2\,^3P_2$	3.929	4.000	3.98	4.02	3.972	3.986	3.98
$3\,^3P_2$		4.400				4.350	4.36
$4\,^3P_2$		4.767				4.786	4.72

into the following equations:

$$M^2_{\psi(4040)} = M^2_{J/\psi} + (3-1)(1+f_{cc}(m_c+m_c))/\alpha'_{c\bar{c}(1)}\,, \tag{9}$$

$$M^2_{\rho(1450)} = M^2_{\rho} + (2-1)(1+f_{nn}(m_u+m_u))/\alpha'_{n\bar{n}(1)}\,, \tag{10}$$

$$M^2_{K^*(1580)} = M^2_{K^*(892)} + (2-1)(1+f_{ns}(m_u+m_s))/\alpha'_{n\bar{s}(1)}\,, \tag{11}$$

and with the help of the following relations derived from (2), (5) and (7):

$$2f_{ns}(m_u+m_s) = f_{nn}(m_u+m_u) + f_{ss}(m_s+m_s)\,, \tag{12}$$

$$2f_{cn}(m_c+m_u) = f_{nn}(m_u+m_u) + f_{cc}(m_c+m_c)\,, \tag{13}$$

$$2f_{cs}(m_c+m_s) = f_{cc}(m_c+m_c) + f_{ss}(m_s+m_s)\,, \tag{14}$$

one can obtain (in GeV^{-1})

$$f_{nn} = 0.601\,,\quad f_{ns} = 0.584\,,\quad f_{cn} = 0.210\,,\quad f_{cs} = 0.235\,,\quad f_{cc} = 0.141\,. \tag{15}$$

Based on the above parameters, we find that $\alpha_{n\bar{n}(1)}(0)-\alpha_{n\bar{n}(2)}(0) = 1.35$, $\alpha_{n\bar{s}(1)}(0)-\alpha_{n\bar{s}(2)}(0) = 1.44$, $\alpha_{c\bar{n}(1)}(0)-\alpha_{c\bar{n}(2)}(0) = 1.41$, $\alpha_{c\bar{s}(1)}(0)-\alpha_{c\bar{s}(2)}(0) = 1.50$, and $\alpha_{c\bar{c}(1)}(0)-\alpha_{c\bar{c}(2)}(0) = 1.47$, which are in good agreement with the quantitative results given by [33, 34]: $\alpha_{i\bar{j}(1)}(0)-\alpha_{i\bar{j}(2)}(0) \approx 1.3$–$1.6$.

The spectrum of $c\bar{c}$ is well predicted by different theoretical approaches, and the excited vector $c\bar{c}$ states are well established experimentally; these predicted and measured results serve as a good testing ground for the question whether our proposed mass relation (7) can give reliable predictions. The measured $1\,^3S_1$, $1\,^3D_1$, $1\,^3P_0$ and $1\,^3P_2$ $c\bar{c}$ masses are used as input,[4] and from (7), (8) and (15), our predicted masses of the radial excitations of the $c\bar{c}$ states lying on the $1\,^3S_1$-like trajectories are shown in Table 2 and Fig. 1. A comparison of results predicted by us and those from experiments and other theoretical approaches is also given in Table 2 and Fig. 1. Clearly, the masses of the $c\bar{c}$ states lying on the $1\,^3S_1$-like trajectories predicted by the mass relation (7) agree remarkably well with those from measurements and other theoretical approaches.

3 Masses of the $c\bar{q}$ states lying on the $1\,^3S_1$-like trajectories

In this section, we shall estimate the masses of the $c\bar{q}$ ground states lying on the $1\,^3S_1$-like trajectories using (4); then the radial excited $c\bar{q}$ masses can be given by (7). In order to derive the masses of the $c\bar{n}$ and $c\bar{s}$ states using (4), we should know the masses of $n\bar{n}$ $(I=1)$, $n\bar{s}$ and $c\bar{c}$. Experimentally, the $n\bar{n}$ $(I=1)$, $n\bar{s}$, $c\bar{c}$ states for the $1\,^3S_1$, $1\,^3P_2$ and $1\,^3D_1$ multiplets are well established [5]; however, for the $1\,^3P_0$ $n\bar{n}$ $(I=1)$, $n\bar{s}$ and $c\bar{c}$ states, only the $c\bar{c}$ state, $\chi_{c0}(1P)$, is well established, and the assignment for $n\bar{n}$ $(I=1)$ and $n\bar{s}$ remains an open question. In the recent literature, there is not yet consensus on the $1\,^3P_0$ $n\bar{n}$ $(I=1)$ mass given by lattice simulations. For example, both $M_{a_0(1\,^3P_0)} \sim 1$ GeV [44–46] and $M_{a_0(1\,^3P_0)} \sim 1.4$–$1.6$ GeV [47–49] have been predicted re-

[4] The $1\,^3D_3$ $c\bar{c}$ mass is given by $\sqrt{\frac{2}{\alpha'_{c\bar{c}(1)}} + M^2_{J/\psi}}$ [32].

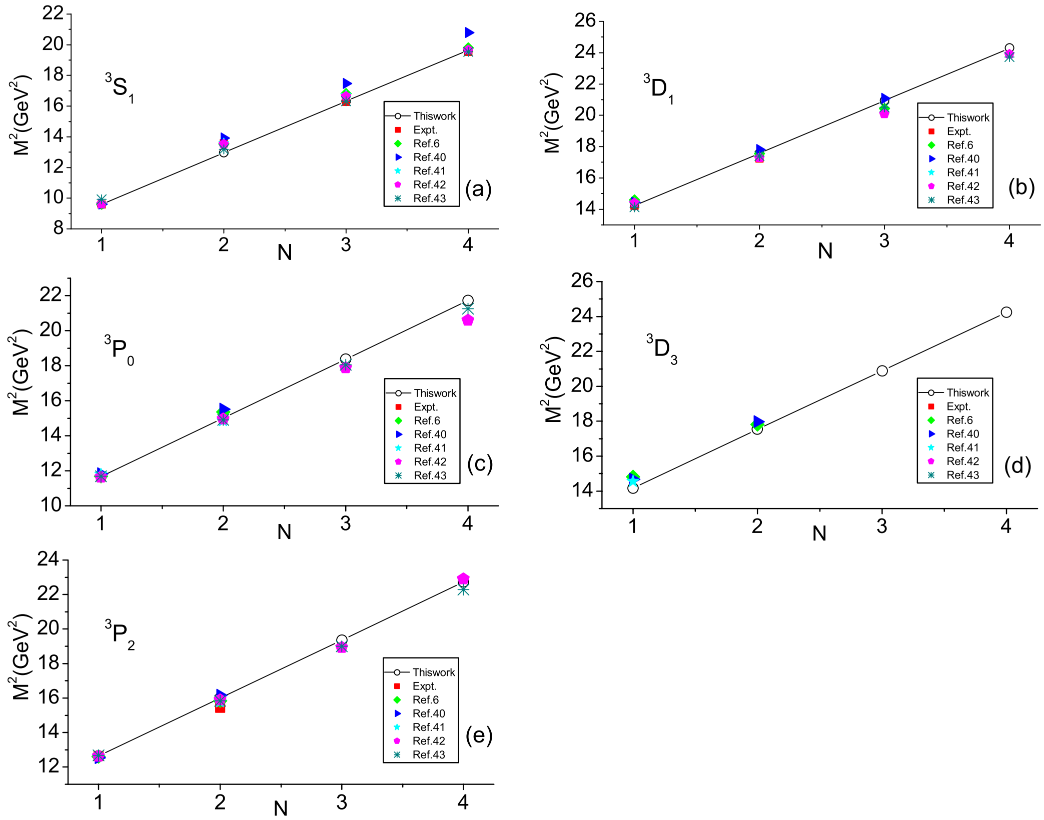

Fig. 1. The (N, M^2)-trajectories for the $c\bar{c}$ states. The predicted masses are displayed numerically in Table 2

cently. In the present work, we shall take $M_{a_0(1\,^3P_0)} = (1.0 + 1.04 + 1.01)/3 \approx 1.02$ GeV, the average value of the predictions given by recent lattice QCD calculations [44–46], taking into consideration that the naive quark model predicts that the spin–orbit force makes $a_0(1\,^3P_0)$ lighter with respect to $a_2(1\,^3P_2)$ ($M_{a_2(1\,^3P_2)} = 1.3183$ GeV [5]), and the same behavior is evident in the $c\bar{c}$ and $b\bar{b}$ spectra [38]. Lattice studies suggest the mass of the $1\,^3P_0$ kaon to be 100–130 MeV heavier than the mass of a_0 [46]. This is not easily related to any current experimental candidate, while it is consistent with the result $M_{n\bar{s}(1\,^3P_0)} = 1090 \pm 40$ MeV from the K matrix analysis of the $K\pi$ S-wave performed by Anisovich and Sarantsev [50]. In this work, we shall take $M_{n\bar{s}(1\,^3P_0)} = 1.09$ GeV.

With the help of Table 1 and the following masses (in GeV) used as input:

$$\begin{aligned}
M_{n\bar{n}(1\,^3S_1)} &= 0.7755\,, & M_{n\bar{n}(1\,^3P_2)} &= 1.3183\,,\\
M_{n\bar{s}(1\,^3S_1)} &= 0.896\,, & M_{n\bar{s}(1\,^3P_2)} &= 1.4324\,,\\
M_{c\bar{c}(1\,^3S_1)} &= 3.096916\,, & M_{c\bar{c}(1\,^3P_2)} &= 3.5562\,,\\
M_{n\bar{n}(1\,^3P_0)} &= 1.02\,, & M_{n\bar{n}(1\,^3D_1)} &= 1.701\,,\\
M_{n\bar{s}(1\,^3P_0)} &= 1.09\,, & M_{n\bar{s}(1\,^3D_1)} &= 1.735\,,\\
M_{c\bar{c}(1\,^3P_0)} &= 3.41476\,, & M_{c\bar{c}(1\,^3D_1)} &= 3.7711\,;
\end{aligned}$$

the masses of the $c\bar{n}$ and $c\bar{s}$ states predicted by (4) and (7) are shown in Table 3 and Figs. 2 and 3, together with the recent predictions by some modified quark models [13, 15, 16]. The masses of the $1\,^3D_3$ $c\bar{n}$ and $c\bar{s}$ states are given by [32]

$$M_{c\bar{n}(1\,^3D_3)} = \sqrt{\frac{2}{\alpha'_{c\bar{n}(1)}} + M^2_{c\bar{n}(1\,^3S_1)}}\,,$$

$$M_{c\bar{s}(1\,^3D_3)} = \sqrt{\frac{2}{\alpha'_{c\bar{s}(1)}} + M^2_{c\bar{s}(1\,^3S_1)}}\,.$$

4 Discussions

From Table 3 and Figs. 2 and 3, it is clear that the agreement between our predicted D and D_s masses and the recent predictions by some modified quark model [13, 15, 16] is quite good. It therefore appears likely that Regge phenomenology is capable of describing the D and D_s masses with reasonable accuracy.

Our predicted D_0 meson mass is 2.268 GeV, in good agreement with the preliminary Belle measurement of $2290 \pm 22 \pm 20$ MeV [51] and the current Belle mass of $2308 \pm 17 \pm 32$ MeV [52] within one to two percent of accuracy, while it is in disagreement with the FOCUS mass of $2407 \pm 21 \pm 35$ MeV [53]. For the D_{s0} mass, our prediction is 2.331 GeV, only 13.7 MeV higher than the experimental result, and in good agreement with the recent result predicted by the use of QCD sum rules that $M_{c\bar{s}(1\,^3P_0)} = 2.31 \pm 0.03$ GeV [54]. Our analysis supports the conclusion that the $D_{s0}(2317)$ can be identified as a conventional $1\,^3P_0$ $c\bar{s}$ state.

Table 3. D and D_s spectra. † Average value of 2290 and 2308 MeV by Belle. All in GeV

D_s	Expt. [5]	This work	[13]	[15]	[16]	D	Expt. [5]	This work	[13]	[15]
$1\,^3S_1$	2.112	2.100	2.110	2.105	2.112	$1\,^3S_1$	2.010	2.010	2.011	2.017
$2\,^3S_1$		2.653			2.711	$2\,^3S_1$		2.540		
$3\,^3S_1$		3.109			3.153	$3\,^3S_1$		2.976		
$1\,^3D_1$		2.775	2.817		2.784	$1\,^3D_1$		2.738	2.762	
$2\,^3D_1$		3.214				$2\,^3D_1$		3.147		
$3\,^3D_1$		3.600				$3\,^3D_1$		3.509		
$1\,^3P_0$	2.3173	2.331	2.325	2.341	2.329	$1\,^3P_0$	2.299†	2.268	2.283	2.260
$2\,^3P_0$		2.839			2.817	$2\,^3P_0$		2.748		
$3\,^3P_0$		3.269			3.219	$3\,^3P_0$		3.156		
$1\,^3D_3$		2.815				$1\,^3D_3$		2.731		
$2\,^3D_3$		3.248				$2\,^3D_3$		3.141		
$3\,^3D_3$		3.630				$3\,^3D_3$		3.504		
$1\,^3P_2$	2.5735	2.562	2.568	2.563	2.577	$1\,^3P_2$	2.459	2.457	2.468	2.493
$2\,^3P_2$		3.032			3.041	$2\,^3P_2$		2.906		
$3\,^3P_2$		3.438			3.431	$3\,^3P_2$		3.295		

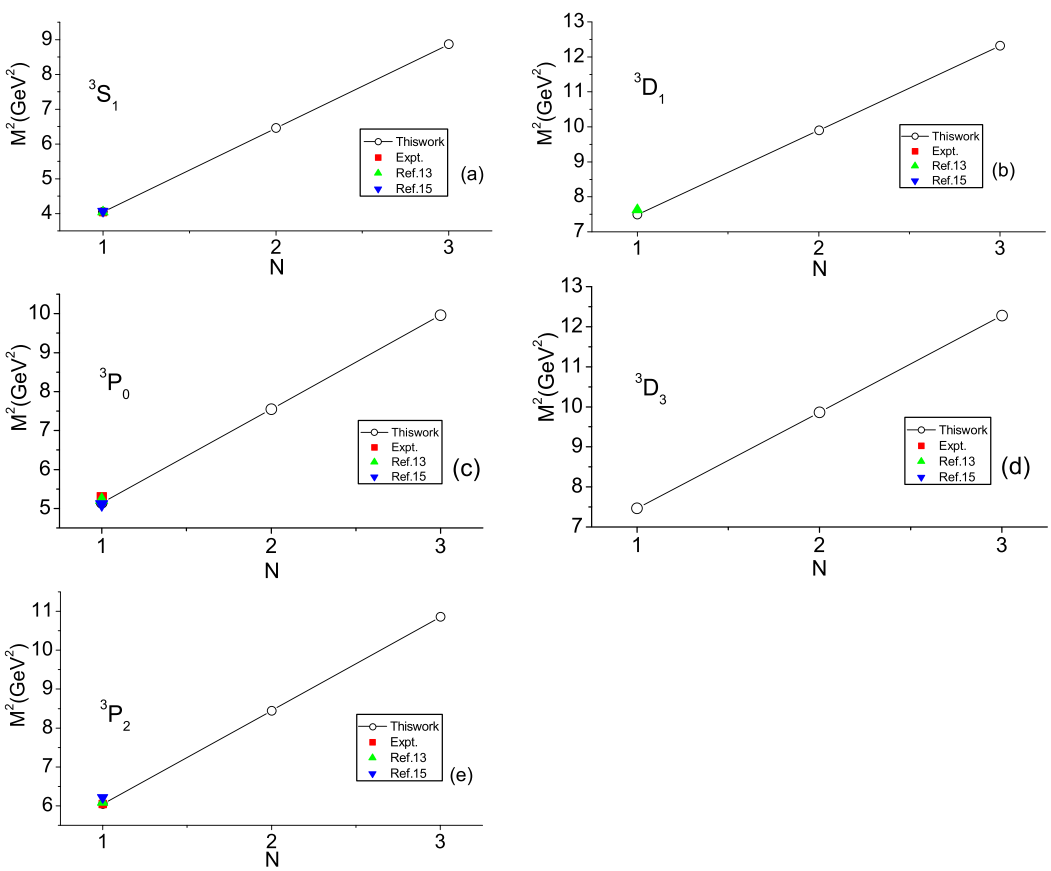

Fig. 2. The (N, M^2)-trajectories for the $c\bar{n}$ states. The predicted masses are displayed numerically in Table 3

Based on the measured masses of the $n\bar{n}$, $n\bar{s}$ and $c\bar{c}$ states for the $1\,^3S_1$ and $1\,^3P_2$ multiplets, the predicted masses for the $1\,^3S_1$ and $1\,^3P_2$ $c\bar{q}$ state by (4) are in excellent agreement with the measurements. Taking $M_{n\bar{n}(1\,^3P_0)} = 1.02$ GeV and $M_{n\bar{s}(1\,^3P_0)} = 1.09$ GeV, (4) gives an accurate determination for the D_0 and D_{s0} masses, and it implies that $a_0(980)$ could indeed be the lightest non-singlet scalar meson.

BaBar recently reported the discovery of a new $D_{sJ}(2860)$ state with a mass of $2856.6 \pm 1.5 \pm 5.0$ MeV and a width of $48 \pm 7 \pm 10$ MeV [17]. BaBar observed this state only in the D^0K^+ or $D^+K^0_S$ system and found no evidence in the $D^{*0}K^+$ and $D^{*+}K^0_S$ channels; therefore, the possible J^P of $D_{sJ}(2860)$ are $0^+, 1^-, 2^+, 3^-, \ldots$

Based on our prediction as shown in Table 3, the $D_{sJ}(2860)$ mass is about 213 MeV higher than the $2\,^3S_1$

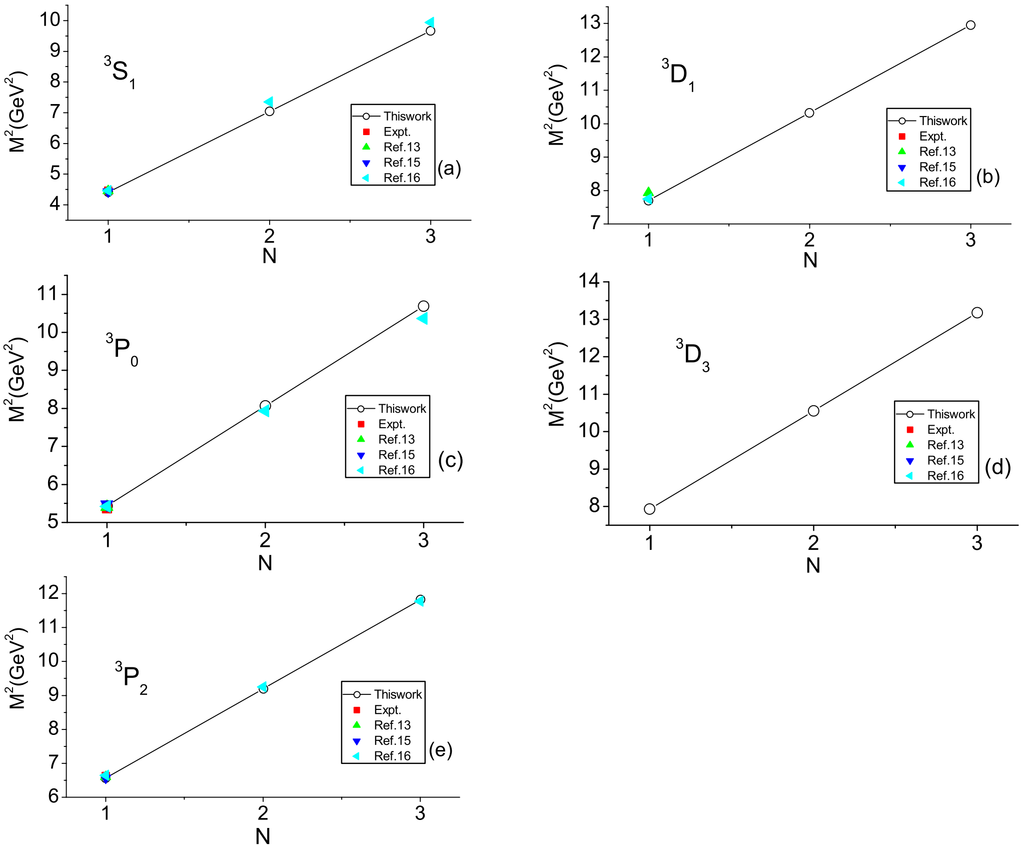

Fig. 3. The (N, M^2)-trajectories for the $c\bar{s}$ states. The predicted masses are displayed numerically in Table 2

$c\bar{s}$ mass (2653 MeV), and 82 MeV higher than the $1\,^3D_1$ $c\bar{s}$ mass (2775 MeV). The decay pattern and total width of $D_{sJ}(2860)$ for various quantum number assignments has been investigated by Zhang et al. [55] in a $^3P_0$ decay model. If $D_{sJ}(2860)$ is the $2\,^3S_1$ $c\bar{s}$ state, its total width is about 90 MeV, and the dominant decay modes are DK^* with $\Gamma(DK^*) = 24$ MeV and D^*K with $\Gamma(D^*K) = 64$ MeV, and the DK mode is only 12 keV [55]. If the $D_{sJ}(2860)$ is the $1\,^3D_1$ $c\bar{s}$ state, its total width is about 132 MeV, and $\Gamma(DK) : \Gamma(D_s\eta) : \Gamma(D^*K) : \Gamma(D_s^*\eta) : \Gamma(DK^*) \approx 42 : 12 : 7 : 1 : 4$ [55]. Both the predicted mass and width for the $2\,^3S_1$ or $1\,^3D_1$ $c\bar{s}$ state are significantly larger than the experimental data of $D_{sJ}(2860)$, making the assignment of $D_{sJ}(2860)$ to the $2\,^3S_1$ or $1\,^3D_1$ $c\bar{s}$ state unfavorable.

However, the $D_{sJ}(2860)$ mass is close to our predicted $2\,^3P_0$ $c\bar{s}$ mass (2839 MeV) and $1\,^3D_3$ $c\bar{s}$ mass (2815 MeV). If $D_{sJ}(2860)$ is the $2\,^3P_0$ $c\bar{s}$ state, its total width becomes around 54 MeV, consistent with the measured width of $D_{sJ}(2860)$, $48 \pm 7 \pm 10$ MeV, within errors, and the dominant decay modes are DK with $\Gamma(DK) = 37$ MeV and $D_s\eta$ with $\Gamma(D_s\eta) = 16$ MeV [55]; the decay modes D^*K, $D_s^*\eta$ and DK^* are forbidden. The suggestion that the $D_{sJ}(2860)$ can be identified as the $2\,^3P_0$ $c\bar{s}$ state has been given by [16,56]. If $D_{sJ}(2860)$ is the $1\,^3D_3$ $c\bar{s}$ state, its total width is about 37 MeV, also compatible with $48 \pm 7 \pm 10$ MeV within errors, and the dominant decay modes are DK with $\Gamma(DK) = 22$ MeV and D^*K with $\Gamma(D^*K) = 13$ MeV [55]. The assignment of $D_{sJ}(2860)$ to the 3^- $c\bar{s}$ state has been proposed by [57]. At present, only the decay $D_{sJ}(2860) \to DK$ is observed experimentally, which is not enough to distinguish the above two possible assignments; therefore, the assignment of $D_{sJ}(2860)$ to the $2\,^3P_0$ or $1\,^3D_3$ $c\bar{s}$ state seems favorable by the available experimental information.

It should be noted that for the $2\,^3P_0$ $c\bar{s}$ state, the decay modes D^*K, $D_s^*\eta$ and DK^* are forbidden, and the $1\,^3D_3$ $c\bar{s}$ state has a large D^*K decay width and a small $D_s\eta$ decay width (~ 1.2 MeV [55]); therefore, a further experimental search of the $D_{sJ}(2860)$ in the DK^*, D^*K, $D_s^*\eta$ and $D_s\eta$ decay modes would certainly be desirable for distinguishing the above two possible assignments.

Our predicted mass of the $2\,^3S_1$ $c\bar{s}$ state is around 2653 MeV, in excellent agreement with the prediction 2658 ± 15 MeV obtained by Chang from the Bethe–Salpeter equation [58], and 48 MeV lighter than the $D_{sJ}(2690)/D_{sJ}(2700)$ mass.[5] If $D_{sJ}(2690)/D_{sJ}(2700)$ is the $2\,^3S_1$ $c\bar{s}$ state, its total width is about 103 MeV in a $^3P_0$ decay model [16], consistent with the measured width of $112 \pm 7 \pm 36$ or $115 \pm 20^{+36}_{-32}$ MeV, and the dominant decay modes are DK with a width of 22 MeV and D^*K with a width of 78 MeV [16]. Also, the $D_{sJ}(2690)/D_{sJ}(2700)$ mass is about 74 MeV lighter than our predicted $1\,^3D_1$ $c\bar{s}$ mass (2775 MeV), if it is the $1\,^3D_1$ $c\bar{s}$ state, and the calculations performed by [55] within a $^3P_0$ model show that its total width is 73 MeV, also roughly consistent with the experimental data, and the dominant decay modes are DK with a width of about 49 MeV and $D_s\eta$ with a width of about 13 MeV. There-

[5] The average value of the BaBar mass and Belle mass is 2701 MeV.

fore, $D_{sJ}(2690)/D_{sJ}(2700)$ as either a $2\,^3S_1$ or a $1\,^3D_1$ $c\bar{s}$ state seems consistent with the experimental data [55]. Considering that the $D_{sJ}(2690)/D_{sJ}(2700)$ mass is about 48 MeV higher than $M_{c\bar{s}(2\,^3S_1)}$, while it is 74 MeV lower than $M_{c\bar{s}(1\,^3D_1)}$, we tend to suggest that $D_{sJ}(2690)/D_{sJ}(2700)$ is most likely a mixture of $D_s(2\,^3S_1)$ and $D_s(1\,^3D_1)$. It has been found that [16] with $2\,^3S_1$ $c\bar{s}$ mixing with $1\,^3D_1$ $c\bar{s}$ (with a mixing angle of approximately -0.5 radians), the total width of $D_{sJ}(2690)/D_{sJ}(2700)$ (with mass set to 2688 MeV) predicted by a $^3P_0$ model becomes about 110 MeV, in good agreement with the data.

5 Summary and conclusion

The masses of the D and D_s states lying on the $1\,^3S_1$-like trajectories are estimated in Regge phenomenology. The predicted masses agree well with the recent results by some modified quark models. It therefore appears likely that Regge phenomenology is capable of describing the masses of D and D_s with reasonable accuracy.

Our predictions show that masses of the recent observed charmed states, such as the $D_{s0}(2317)$, $D_{sJ}(2860)$ and $D_{sJ}(2690)/D_{sJ}(2700)$, can be reasonably reproduced in the simple quark–antiquark picture. Based on our analysis, we suggest that $D_{sJ}(2317)$ can be identified as the conventional $1\,^3P_0$ $c\bar{s}$ state, and the assignment of $D_{sJ}(2860)$ to the $D_s(2\,^3P_0)$ or $D_s(1\,^3D_3)$ states seems favorable, and $D_{sJ}(2690)/D_{sJ}(2700)$ is most likely a mixture of $D_s(2\,^3S_1)$ and $D_s(1\,^3D_1)$.

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